Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

TCS455: Midterm Examination (Set I)

Solution

COURSE : TCS455 (Mobile Communications)
DATE : December 22, 2009
SEMESTER : 2/2009
INSTRUCTOR: Dr. Prapun Suksompong
TIME : 9:00-12:00
PLACE : BKD 26XX

Name	Prapun	ID	555
		Seat	

Instructions:

1. Including this cover page, this exam has 9 pages.
2. **Read these instructions and the questions carefully**.
3. Closed book. Closed notes.
4. Basic calculators are permitted, but borrowing is not allowed.
5. Allocate your time wisely. Some easy questions give many points.
6. Do not cheat. The use of communication devices including mobile phones is prohibited in the examination room.
7. (1 pt) Write your **first name and the <u>last three digits</u> of your ID** on each page of your examination paper, starting from page 2.
8. Your score depends strongly on your explanation of your answer. If the explanation is incomplete, zero score may be given even when the final answer is correct.
9. Do not panic.
10. Dr. Prapun will visit each exam room regularly. In general, there is no need to ask the proctor to call for Dr. Prapun.

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1. (8 pt) If 30 MHz of total spectrum is allocated for a **duplex** wireless cellular system and each **simplex** channel has 10 kHz RF bandwidth, find:

   a. the number of duplex channels.

1 duplex channel = 2 simplex channels

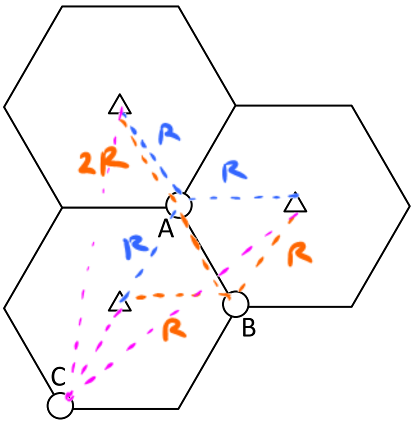
$$\frac{30 \times 10^6}{2 \times (10 \times 10^3)} = 1{,}500 \text{ duplex channels.}$$

   b. the total number of channels per cell site, if N = 3 cell reuse is used.

$$\frac{1{,}500}{3} = 500 \text{ channels per cell site}$$

(1500 channels in part (a) are allocated to each cluster. Each cluster has N=3 cells.)

2. (15 pt) In this question, we consider the SIR value when the cluster size N = 1. Sectoring is not used. Suppose this cellular system has only three base stations. They are marked by triangles located at the centers of three cells in the figure below. Assume that they transmit the same power level.



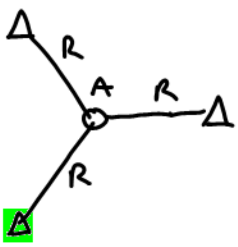
First note that the distance from each user to the associated BS is R.

User A (mobile station A), user B (mobile station B), and user C (mobile station C) are currently associated with the lower-left base station. The locations of these users are marked by the circles in the figure above. Assume a path loss exponent of $\gamma = 4$. Do not approximate distance values.

   a. Find the signal-to-interference ratio (in dB) for user A.

The distance from A to all BS = R.

$$\text{So,} \quad SIR = \frac{P_r}{\sum P_i} = \frac{1/R^\gamma}{\frac{1}{R^\gamma} + \frac{1}{R^\gamma}} = \frac{1}{2} = -3 \text{ dB}$$



Note that we have only two interferers.

Note also that the signals from B and C will not interfere with the signal from A because they use different freq.

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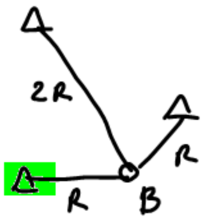
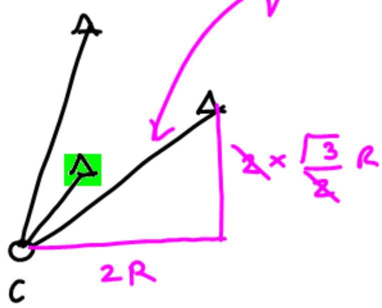
b. Find the signal-to-interference ratio (in dB) for user B.

$$SIR = \frac{\frac{1}{R^\gamma}}{\frac{1}{R^\gamma}+\frac{1}{(2r)^\gamma}} = \frac{1}{1+\frac{1}{2^\gamma}} \underset{\gamma=4}{=} \frac{1}{1+\frac{1}{16}} = -0.263\text{ dB}$$



c. Find the signal-to-interference ratio (in dB) for user C.

$$\sqrt{2^2+\sqrt{3}^2}\,R = \sqrt{7}\,R$$



$$SIR = \frac{\frac{1}{R^\gamma}}{2\times\frac{1}{(\sqrt{7}R)^\gamma}} = \frac{\sqrt{7}^\gamma}{2} \underset{\gamma=4}{=} \frac{49}{2} = 13.9\text{ dB}$$

3. (15 pt) A cellular service provider decides that its users can tolerate a signal-to-interference ratio of 20 dB in the worst case. Assume a path loss exponent of $\gamma = 4$. You may approximate the distances between the users and the interfering co-channel base stations by the center-to-center co-channel distance D.

$$20\text{ dB} = 10^{20/10} = 10^2 = 100$$

a. Find the optimal value of cluster size N for omnidirectional antennas.

$$SIR = \frac{P_r}{\sum_{i=1}^{K} P_i} = \frac{P_r}{K P_i} = \frac{\cancel{k} R^{-\gamma}}{K \cancel{k} D^{-\gamma}} = \frac{1}{K}\left(\frac{D}{R}\right)^\gamma = \frac{1}{K}\left(\sqrt{3N}\right)^\gamma \underset{\gamma=4}{=} \frac{1}{K} 9N^2 \geq 100$$

K = 6 for omnidirectional antennas

$$\Rightarrow N \geq \sqrt{\frac{100K}{9}} = 10\times\sqrt{\frac{2}{3}} = 8.165 \longrightarrow \text{Optimal } N = 9$$

b. Find the optimal value of cluster size N for 120° sectoring.

$$N \geq \sqrt{\frac{100K}{9}} = 4.714 \longrightarrow \text{Optimal } N = 7$$

K = 2 for 120° sectoring

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c. Find the optimal value of cluster size N for 60° sectoring.

$$N \geq \sqrt{\frac{100k}{9}} = \frac{10}{3} = 3.33 \longrightarrow \text{Optimal } N = 4$$

$k=1$ for 60° sectoring

m=2

4. (5 pt) There are 1000 users subscribed to a cellular system. The call request rate for each user is 2 call requests per week. For each call, the average call duration is 1 min. If the system has only two channels and it is used to support the whole 1000 users, what is the blocking probability?

$$\lambda = 2 \times 1{,}000 \ \frac{\text{requests}}{\text{week}} \times \frac{1 \text{ week}}{7 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hrs.}} \times \frac{1 \text{ hr}}{60 \text{ min}}$$

$$\frac{1}{\mu} = 1 \text{ min.}$$

$$A = \frac{\lambda}{\mu} = \frac{2 \times 1000}{7 \times 24 \times 60} = 0.198$$

$m = 2$

Erlang B formula $\rightarrow P_b = \dfrac{A^2/2}{1 + A + \frac{A^2}{2}} = 0.016 = 1.6\%$

5. A function $\text{ErlangB}(m, A)$ is defined by

If you don't remember the formula, it is given in the next question!!

$$\text{ErlangB}(m, A) = \frac{\dfrac{A^m}{m!}}{\displaystyle\sum_{k=0}^{m} \frac{A^k}{k!}}$$

a. (4 pt) Why is this function useful? How can we use it in designing cellular system? Your score depends on the completeness, correctness, and clarity of your answer.

For a cell (or a sector) in a cellular system which has m channels and the amount of traffic is A Erlang, this function is directly used to determine the probability Pb that call requests will be blocked by the system because all channels are currently used. The amount of traffic (A) can be found by the product of the call request rate and the average call duration. When we design a cellular system, the blocking probability Pb should be less than some pre-determined value. In which case, the funtion above can be used to suggest the minimum number of channels per cell (or sector). If we already know the number of channels per cell (or sector) of the system, the function can also be used to determined how many users the system can support as well.

Blocking probability Pb and the SIR values determine the quality of service for cellular users. We can use Pb to trade with SIR and vice versa.

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m A m A

b. (2 pt) Compare $\text{ErlangB}(1000, 2000)$ and $\text{ErlangB}(1000, 2001)$.
Which one is larger?

more Erlang
⇒ more traffic
⇒ larger P_b

m A m A

c. (2 pt) Compare $\text{ErlangB}(1000, 2000)$ and $\text{ErlangB}(1001, 2000)$.
Which one is larger?

less # channels
⇒ larger P_b

d. (1 pt) Suppose

$$\text{ErlangB}(1000, A_1) = \text{ErlangB}(2000, A_2) = \text{ErlangB}(3000, A_3) = 0.05$$

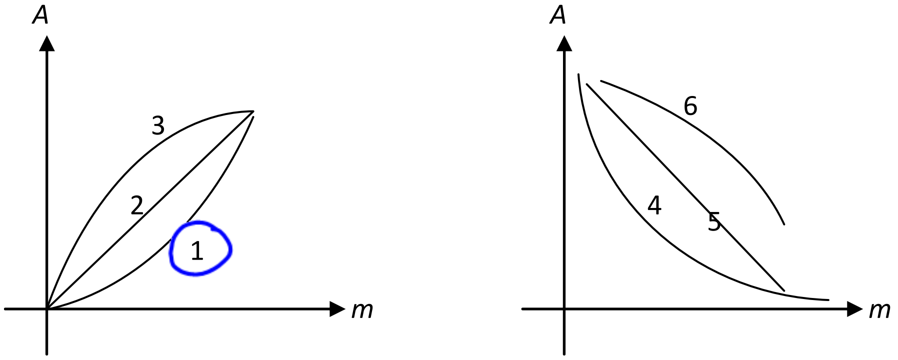
Let $A_4 = A_1 + A_2$.

Compare A_3 and A_4. Which one is larger? A_3

3,000 channels are divided into two groups of size 1,000 and 2,000.

So, the trunking efficiency should be higher for the 3000 channels case; that is it should be able to support more traffic for the same blocking probability.

e. (1 pt) Consider the relation $\text{ErlangB}(m, A) = 0.005$. Which plot below best represents the relationship between m and A?



Hint: No calculation is needed. Think about what the formula says.

We know that as we increase m, the increased # channel allows us to support more traffic (A). So, the graph should be an increasing function.

Now, suppose we increase m by 2x, we know, by the increased trunking efficiency, that A should be increased by more than 2x.

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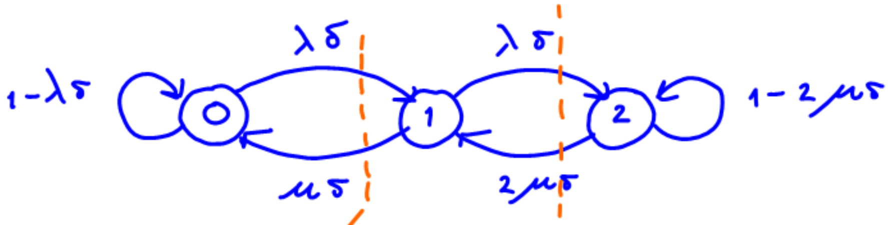
6. (15 pt) Consider a system which has 2 channels. We would like to find the blocking probability via the Markov chain method. Assume that the total call request rate is 30 calls per hour and the average call duration is 6 mins.

   a. **<u>Draw the Markov chain</u>** via discrete time approximation. Assume that the duration of each time slot is 1 millisecond. Don't forget to indicate the transition probabilities on the arrows.

$$\frac{1}{\mu} = 6 \text{ min} = \frac{1}{10} \text{ hr.}$$



$$\lambda\delta = 30 \times \frac{1\times 10^{-3}}{3600} = 8.33\times 10^{-6}$$

$$\mu\delta = 10 \times \frac{1\times 10^{-3}}{3600} = 2.78 \times 10^{-6}$$

   b. Find the long-term blocking probability **<u>from the Markov chain</u>**.

$$A = \frac{\lambda}{\mu} = 30 \times \frac{1}{10} = 3 \text{ Erlangs}$$

$$P_0 \lambda\cancel{\delta} = P_1 \mu \cancel{\delta} \qquad P_1 \lambda\delta = P_2 \times 2\mu\delta$$

$$P_1 = A P_0 \qquad P_2 = \frac{1}{2} A P_1 = \frac{1}{2} A^2 P_0$$

$$P_0 + P_1 + P_2 = 1 \longrightarrow P_0 = \frac{1}{1 + A + \frac{A^2}{2}} \Rightarrow P_2 = \frac{A^2/2}{1 + A + \frac{A^2}{2}} = \frac{9}{17} = 0.529$$

$$P_2 = P_b$$

   c. **<u>Use Erlang B formula</u>**, find the blocking probability.

same calculation

$A = 3,\ m = 2$

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=1}^{m} \frac{A^i}{i!}} = \frac{3^2/2}{1 + 3 + 3^2/2} = \frac{9}{2+6+9} = \frac{9}{17} = 0.529$$

7. (5 pt) What do we mean when we say a Markov chain is memoryless? Your score depends on the completeness, correctness, and clarity of your answer.

The future is conditionally independent of the past history, given the present state. In other words, the entire past history is summarized in the present state. We need no memory of the past history (only the present) to probabilistically predict the future.

This memoryless property is also referred to as the Markov property, after A.A. Markov who first described and studied this class of processes in the early 1900s.

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8. (14 pt) Complete the following M/M/m/m queue description with the following terms:

(I) Bernoulli (II) binomial (III) exponential
(IV) Gaussian (V) geometric (VI) Poisson

We haven't assumed that these intervals are small!

The Erlang B formula is derived under some assumptions. Two important assumptions are (1) the call request process is modeled by a/an Poisson (A)________ process and (2) the call durations are assumed to be i.i.d. exponential (B)________ random variables. For the call request process, if we consider non-overlapping time intervals, the numbers of call requests in these intervals are i.i.d. ____Poisson (C)________ random variables. The times between adjacent call requests are i.i.d. ____exponential (G)____ random variables.

In order to analyze or simulate the system described above, we consider slotted time where the length of each time slot is small. This technique shifts our focus from continuous-time Markov chain to discrete-time Markov chain. In the limit, only one of the two events can happen during any particular slot: either (1) there is one new call request or (2) there is no new call request. So, the numbers of new call requests in the slots can be described by i.i.d. Bernoulli (D)________ random variables. In which case, if we count the total number of call requests during *n* slots, we will get a/an ____binomial (E)________ random variable because it is a sum of i.i.d. ____Bernoulli (D)________ random variables.

This means under the Bernoulli assump.

When we consider a particular time interval *I* (not necessarily small), the number of slots in this interval will increase as the slots get smaller. In the limit, the number of call requests in the time interval *I* which we approximated by a ____binomial (E)________ random variable before will approach a/an Poisson (C)________ random variable.

number of anything is a discrete r.v.!

Similarly, if we consider the numbers of slots between adjacent call requests, these number will be i.i.d. ____geometric (F)________ random variables. These random variables can be thought of as discrete counterparts of the i.i.d. exponential (G)________ random variables in the continuous-time model.

Put your answers in the box below. Some terms above are used more than once. Some terms are not used.

(A)	VI
(B)	III
(C)	VI
(D)	I
(E)	II
(F)	V
(G)	III

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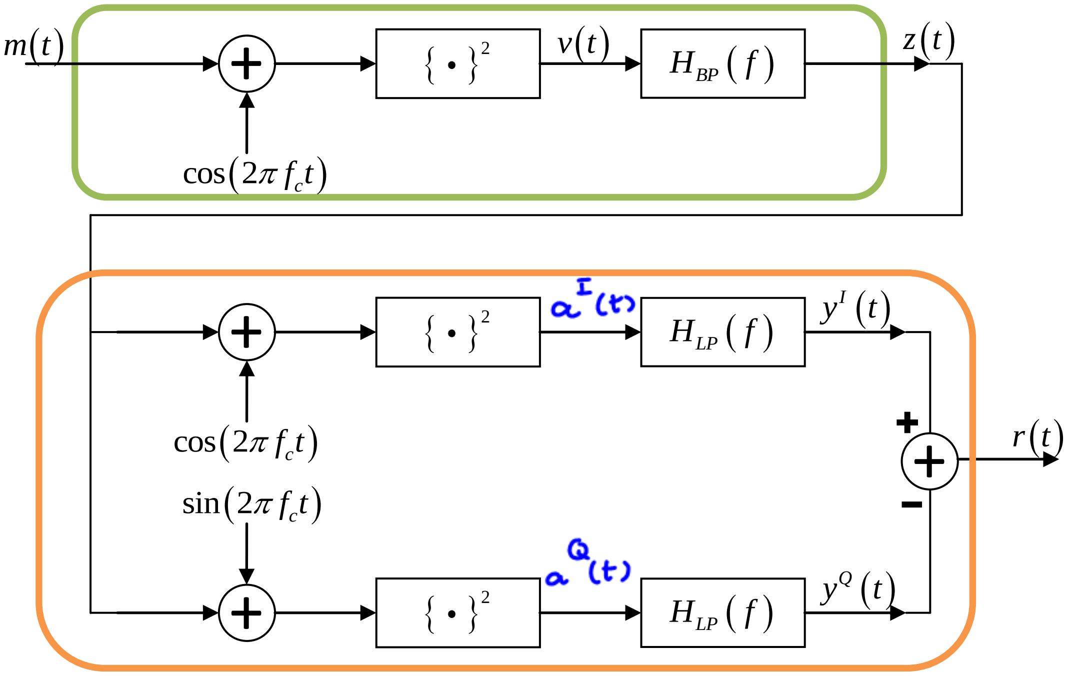
9. Suppose $m(t) \xrightarrow{\mathcal{F}} M(f)$ is bandlimited to W, i.e., $|M(f)| = 0$ for $|f| > W$. Consider the following block diagram:



where $f_c \gg W$,

Let $\omega_c = 2\pi f_c$

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le W \\ 1, & |f + f_c| \le W \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad H_{LP}(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$$

a. (5 pt) Give a simplified formula for $z(t)$.

$$v(t) = (m(t) + \cos(\omega_c t))^2 = \underbrace{\cancel{m^2(t)}}_{BPF} + 2m(t)\cos(\omega_c t) + \cos^2(\omega_c t)$$

$$\downarrow$$

$$\underbrace{\cancel{\tfrac{1}{2}}}_{BPF} + \tfrac{1}{2}\underbrace{\cancel{\cos(2\omega_c t)}}_{BPF}$$

$$z(t) = 2m(t)\cos(\omega_c t)$$

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b. (5 pt) Give a simplified formula for $r(t)$.

$$a^I(t) = (z(t) + \cos w_c t)^2 = (2m(t)\cos w_c t + \cos w_c t)^2 = \cos^2 w_c t\,(2m(t)+1)^2$$

$$= \Big(\tfrac{1}{2} + \underbrace{\tfrac{1}{2}\cos(2w_c t)}_{\text{LPF}}\Big)\Big(\underbrace{4m^2(t) + 4m(t) + 1}_{\text{baseband}}\Big)$$

$$y^I(t) = \text{LPF}\{2m^2(t)\} + 2m(t) + \tfrac{1}{2}$$

$$a^Q(t) = (z(t) + \sin w_c t)^2 = (2m(t)\cos w_c t + \sin w_c t)^2$$

$$= 4m^2(t)\cos^2 w_c t + 4m(t)\cos(w_c t)\sin(w_c t) + \sin^2 w_c t$$

$$= 2m^2(t) + \underbrace{2m^2(t)\cos(2w_c t)}_{\text{LPF}} + \underbrace{m(t)\sin(2w_c t)}_{\text{LPF}} + \tfrac{1}{2} - \underbrace{\tfrac{1}{2}\cos(2w_c t)}_{\text{LPF}}$$

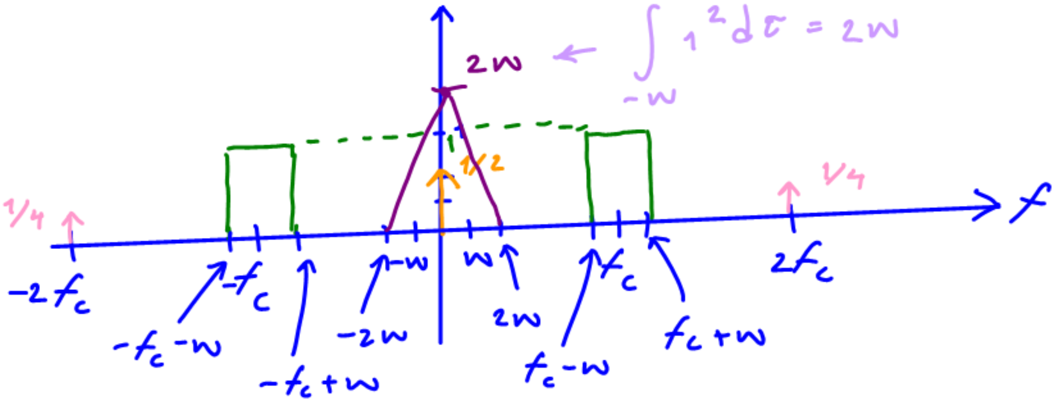
$$y^Q(t) = \text{LPF}\{2m^2(t)\} + \tfrac{1}{2}$$

$$r(t) = y^I(t) - y^Q(t) = 2m(t)$$

c. (2 pt) Suppose $M(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$

Plot $|V(f)|$. Your score depends strongly on your explanation and how accurate your plot is.

From (a), $v(t) = m^2(t) + 2m(t)\cos(w_c t) + \tfrac{1}{2} + \tfrac{1}{2}\cos(2w_c t)$



$2\cos^2 x = 1 + \cos(2x)$

$2\sin^2 x = 1 - \cos(2x)$

$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$

$\cos(2\pi f_c t + \theta) \xrightarrow{\mathcal{F}} \frac{1}{2}\delta(f - f_c)e^{j\theta} + \frac{1}{2}\delta(f + f_c)e^{-j\theta}$

$g(t - t_0) \xrightarrow{\mathcal{F}} e^{-j2\pi f t_0} G(f)$

$e^{j2\pi f_0 t} g(t) \xrightarrow{\mathcal{F}} G(f - f_0)$

$g(t)\cos(2\pi f_c t) \xrightarrow{\mathcal{F}} \frac{1}{2}G(f - f_c) + \frac{1}{2}G(f + f_c)$

Recall: convolution

$$\{x * y\}(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$$

$$\{x * y\}(0) = \int_{-\infty}^{\infty} x(\tau) y(-\tau) d\tau$$